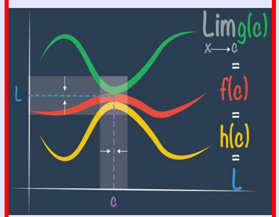
Math 261

Spring 2023

Lecture 53



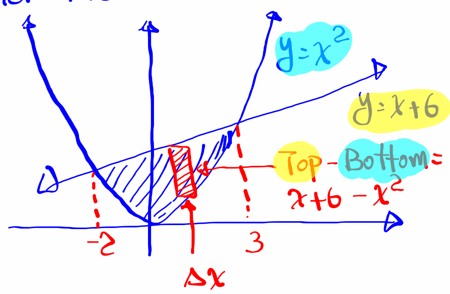
Feb 19-8:47 AM

Class QZ 14

Find the area shaded below.

Exact Answer only

Hint: Solve $x^2 = x + 6$ to find intersection Points.



$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x = 3, \ x = -2$

$$A = \int_{-2}^{3} (x + 6 - x^2)\,dx = \left(\frac{x^2}{2} + 6x - \frac{x^3}{3}\right)\Bigg]_{-2}^{3}$$

$$= \left(\frac{9}{2} + 18 - \frac{27}{3}\right) - \left(\frac{4}{2} - 12 + \frac{8}{3}\right)$$

$$= \frac{5}{2} + 30 - \frac{35}{3} = \frac{15 + 180 - 70}{6}$$

$$= \boxed{\frac{125}{6}}$$

May 18-9:33 AM

If f is cont. on $[a,b]$ and

$g(x) = \int_a^x f(t)\,dt \qquad a \le x \le b$

then $g'(x) = f(x)$ and $g(x)$ is diff. on (a,b)

and $g(x)$ is cont. on $[a,b]$

$$g(x) = \int_{u(x)}^{v(x)} f(t)\,dt$$

$$g'(x) = f(v(x)) \cdot v'(x) - f(u(x)) \cdot u'(x)$$

$$g(x) = \int_{4x}^{x^2} \sin^2 t \; dt$$

$$g'(x) = \sin^2 x^2 \cdot 2x \quad - \sin^2 4x \cdot 4$$

$$\boxed{g'(x) = 2x \sin^2 x^2 - 4 \sin^2 4x}$$

May 22-8:55 AM

find $\dfrac{d}{dx} \displaystyle\int_{2x}^{3x} \frac{t^2 - 1}{t^2 + 1}\,dt$

$3x$: $v(x)$

$\frac{t^2-1}{t^2+1}$: $f(t)$

$2x$: $u(x)$

$$= f(v(x)) \cdot v'(x) \quad - f(u(x)) \cdot u'(x)$$

$$= \frac{(3x)^2 - 1}{(3x)^2 + 1} \cdot 3 - \frac{(2x)^2 - 1}{(2x)^2 + 1} \cdot 2$$

$$= \frac{9x^2 - 1}{9x^2 + 1} \cdot 3 - \frac{4x^2 - 1}{4x^2 + 1} \cdot 2$$

$$= \frac{(4x^2+1) \cdot 3(9x^2-1) - (9x^2+1) \cdot 2(4x^2-1)}{(9x^2+1)(4x^2+1)} = \boxed{}$$

May 22-9:02 AM

$$g(x) = \int_{\tan x}^{x^2} \underbrace{\frac{1}{\sqrt{2+t^4}}}_{f(t)}\,dt$$

$u(x) = \tan x$

$v(x) = x^2$

$$= f(v(x))\cdot v'(x) - f(u(x))\cdot u'(x)$$

$$= \frac{1}{\sqrt{2+(x^2)^4}}\cdot 2x - \frac{1}{\sqrt{2+\tan^4 x}}\cdot \sec^2 x$$

May 22-9:07 AM

If $f(x) = \int_0^x (1-t^2)\cos^2 t\,dt$, on what interval $f(x)$ is increasing?

$f'(x) > 0$

$$f'(x) = \frac{d}{dx}\int_{0 \leftarrow u(x)}^{x \leftarrow v(x)} \underbrace{(1-t^2)\cos^2 t}_{g(t)}\,dt$$

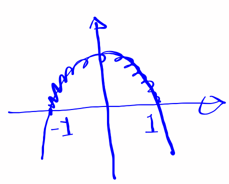
$$= g(v(x))\cdot v'(x) - g(u(x))\cdot u'(x)$$

$$= (1-x^2)\cdot\cos^2 x \cdot 1 - \cancel{(1-0^2)\cos 0\cdot 0}^{\,0}$$

$f'(x) = (1-x^2)\cdot\cos^2 x$

$f'(x) > 0 \qquad (1-x^2)\cdot\cos^2 x > 0 \quad (\cos^2 x \to +)$

$1-x^2>0 \Rightarrow (-1, 1)$



$f(x)$ is increasing on $(-1,1)$.

May 22-9:10 AM

Discuss concavity for $y=\int_0^x \frac{t^2}{t^2+t+2}\,dt$

$y''>0$ C.U.

$y''<0$ C.D.

$$y=\int_0^x \frac{t^2}{t^2+t+2}\,dt \Rightarrow y'=\frac{x^2}{x^2+x+2}\cdot 1-\cancel{\frac{0^2}{0^2+0+2}}\cdot 0 \rightarrow 0$$

$$y'=\frac{x^2}{x^2+x+2}$$

$$y''=\frac{2x(x^2+x+2)-x^2\cdot(2x+1)}{(x^2+x+2)^2}$$

$$y''=\frac{\cancel{2x^3}+2x^2+4x-\cancel{2x^3}-x^2}{(x^2+x+2)^2}=\frac{x^2+4x}{(x^2+x+2)^2}$$

$+$

$y''=0 \rightarrow x^2+4x=0 \rightarrow x=0,\ x=-4$

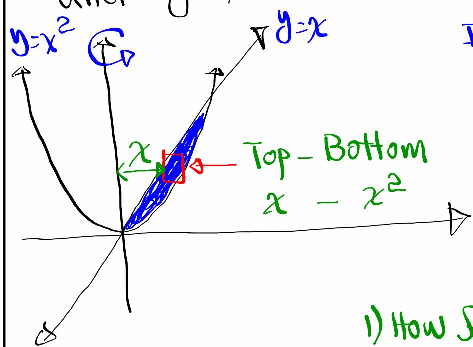
$y''>0$		$y''<0$		$y''>0$
C.U.	-4	C.D.	0	C.U.

Concave up $(-\infty,-4),(0,\infty)$

Concave down $(-4,0)$

May 22-9:17 AM

Find the volume of a solid obtained by rotating the enclosed region by $y=x$ and $y=x^2$ about Y-axis.



Intersection points are $(0,0)$, $(1,1)$

Ref. Rect. is parallel to A.O.R.

Shell Method

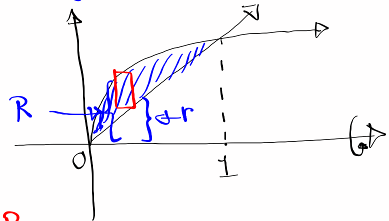
1) How far is Ref. Rect. from A.O.R.? x

2) Height of Ref. Rect. $x-x^2$

$$V=\int_0^1 2\pi\cdot x\cdot(x-x^2)\,dx=\boxed{}$$

May 22-9:25 AM

Find the volume obtained by rotating enclosed region by $y=\sqrt{x}$, $y=x$ about x-axis.



$R=\sqrt{x}$

$r=x$

Ref. Rect. ⊥ A.O.R.

Region not completely attached To A.O.R.

washer method

$$V=\int_0^1 \pi\left[R^2-r^2\right]dx=\pi\int_0^1\left((\sqrt{x})^2-x^2\right)dx$$

$$=\pi\int_0^1(x-x^2)dx$$

$$=\pi\left[\frac{x^2}{2}-\frac{x^3}{3}\right]_0^1$$

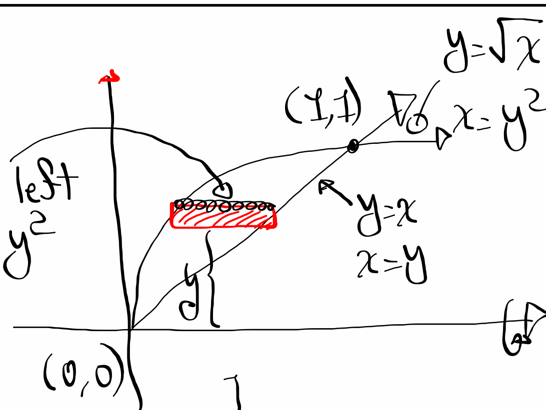
$$=\boxed{\frac{\pi}{6}}$$

May 22-9:31 AM

Let's do Shell Method

Right − left

$y - y^2$



$$V=\int_0^1 2\pi\cdot y\cdot(y-y^2)dy=2\pi\int_0^1(y^2-y^3)dy$$

$$=2\pi\left[\frac{y^3}{3}-\frac{y^4}{4}\right]_0^1$$

$$=2\pi\cdot\frac{1}{12}=\boxed{\frac{\pi}{6}}$$

May 22-9:37 AM

Class QZ 15

Find f_{ave} for $f(x) = \sin 4x$ on $[-\pi, \pi]$.

$$f_{ave} = \frac{1}{\pi-(-\pi)}\int_{-\pi}^{\pi} \sin 4x\,dx = \frac{1}{2\pi}\int_{-\pi}^{\pi}\sin 4x\,dx$$

$u = 4x$

$du = 4dx$

$\frac{du}{4} = dx$

$$= \frac{1}{2\pi}\left[\frac{-1}{4}\cos 4x\right]_{-\pi}^{\pi} = \frac{-1}{8\pi}\left[\cos 4\pi - \cos(-4\pi)\right]$$

$$= \frac{-1}{8\pi}\left[1 - 1\right] = \boxed{0}$$

May 22-9:40 AM